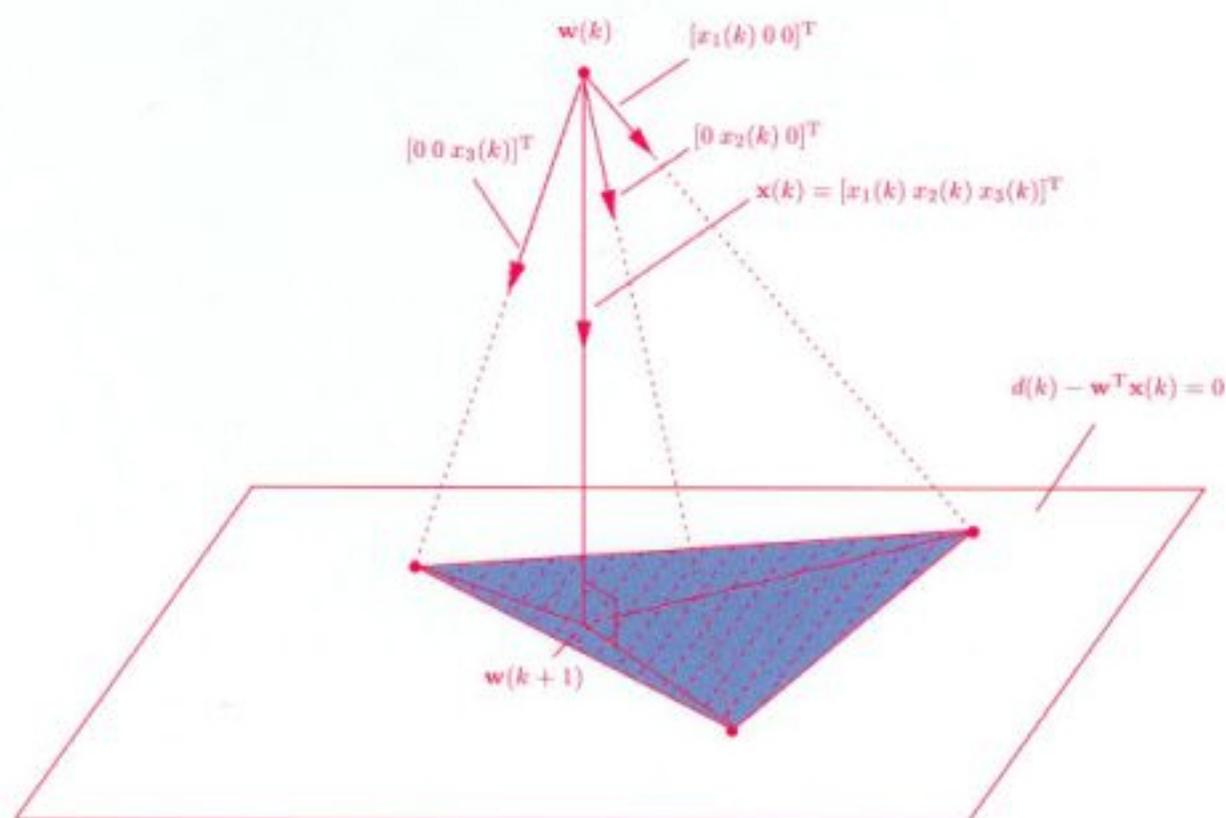


# ADAPTIVE FILTERING

## Algorithms and Practical Implementation

*Second Edition*



**Paulo S. R. Diniz**



**Kluwer Academic Publishers**

# **Adaptive Filtering Algorithms and Practical Implementation**

**Third Edition**

Paulo S.R. Diniz

# Adaptive Filtering Algorithms and Practical Implementation

Third Edition

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## **Note to Instructors**

For the instructors this book has a solution manual for the problems written by Dr. L. W. P. Biscainho available from the publisher. Also available, upon request to the author, is a set of master transparencies as well as the MATLAB<sup>®1</sup> codes for all the algorithms described in the text.

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<sup>1</sup>MATLAB is a registered trademark of The MathWorks, Inc.

To: My Parents,

Mariza,

Paula,

and Luiza.

---

# PREFACE

The field of *Digital Signal Processing* has developed so fast in the last three decades that it can be found in the graduate and undergraduate programs of most universities. This development is related to the increasingly available technologies for implementing digital signal processing algorithms. The tremendous growth of development in the digital signal processing area has turned some of its specialized areas into fields themselves. If accurate information of the signals to be processed is available, the designer can easily choose the most appropriate algorithm to process the signal. When dealing with signals whose statistical properties are unknown, fixed algorithms do not process these signals efficiently. The solution is to use an adaptive filter that automatically changes its characteristics by optimizing the internal parameters. The adaptive filtering algorithms are essential in many statistical signal processing applications.

Although the field of adaptive signal processing has been subject of research for over four decades, it was in the eighties that a major growth occurred in research and applications. Two main reasons can be credited to this growth, the availability of implementation tools and the appearance of early textbooks exposing the subject in an organized manner. Still today it is possible to observe many research developments in the area of adaptive filtering, particularly addressing specific applications. In fact, the theory of linear adaptive filtering has reached a maturity that justifies a text treating the various methods in a unified way, emphasizing the algorithms suitable for practical implementation. This text concentrates on studying on-line algorithms, those whose adaptation occurs whenever a new sample of each environment signal is available. The so-called block algorithms, those whose adaptation occurs when a new block of data is available, are also included using the subband filtering framework. Usually, block algorithms require different implementation resources than the on-line algorithms. This edition also includes basic introductions to nonlinear adaptive filtering and blind signal processing as natural extensions of the algorithms treated in the earlier chapters. The understanding of the introductory material presented is fundamental for further studies in these fields which are described in more detail in some specialized texts.

The idea of writing this book started while teaching the adaptive signal processing course at the graduate school of the Federal University of Rio de Janeiro (UFRJ). The request of the students to cover as many algorithms as possible made me think how to organize this subject such that not much time is lost in adapting notations and derivations related to different algorithms. Another common question was which algorithms really work in a finite-precision implementation. These issues led me to conclude that a new text on this subject could be written with these objectives in mind. Also, considering that most graduate and undergraduate programs include a single adaptive filtering course, this book should not be lengthy. Another objective to seek is to provide an easy access to the working algorithms for the practitioner.

It was not until I spent a sabbatical year and a half at University of Victoria, Canada, that this project actually started. In the leisure hours, I slowly started this project. Parts of the early chapters of this book were used in short courses on adaptive signal processing taught at different institutions, namely: Helsinki University of Technology, Espoo, Finland; University Menendez Pelayo in Seville, Spain; and at the Victoria Micronet Center, University of Victoria, Canada. The remaining parts of the book were written based on notes of the graduate course in adaptive signal processing taught at COPPE (the graduate engineering school of UFRJ).

The philosophy of the presentation is to expose the material with a solid theoretical foundation, while avoiding straightforward derivations and repetition. The idea is to keep the text with a manageable size, without sacrificing clarity and without omitting important subjects. Another objective is to bring the reader up to the point where implementation can be tried and research can begin. A number of references are included at the end of the chapters in order to aid the reader to proceed on learning the subject.

It is assumed the reader has previous background on the basic principles of digital signal processing and stochastic processes, including: discrete-time Fourier- and  $Z$ -transforms, finite impulse response (FIR) and infinite impulse response (IIR) digital filter realizations, multirate systems, random variables and processes, first- and second-order statistics, moments, and filtering of random signals. Assuming that the reader has this background, I believe the book is self contained.

Chapter 1 introduces the basic concepts of adaptive filtering and sets a general framework that all the methods presented in the following chapters fall under. A brief introduction to the typical applications of adaptive filtering are also presented.

In Chapter 2, the basic concepts of discrete-time stochastic processes are reviewed with special emphasis to the results that are useful to analyze the behavior of adaptive filtering algorithms. In addition, the Wiener filter is presented, establishing the optimum linear filter that can be sought in stationary environments. Appendix A briefly describes the concepts of complex differentiation mainly applied to the Wiener solution. The case of linearly constrained Wiener filter is also discussed, motivated by its wide use in antenna array processing. The transformation of the constrained minimization problem into an unconstrained one is also presented. The concept of mean-square error surface is then introduced, another useful tool to analyze adaptive filters. The classical Newton and steepest-descent algorithms are briefly introduced. Since the use of these algorithms would require a complete knowledge of the stochastic environment, the adaptive filtering algorithms introduced in the following chapters come into play. Practical applications of the adaptive filtering algorithms are revisited in more detail at the end of Chapter 2 where some examples with closed form solutions are included in order to allow the correct interpretation of what is expected from each application.

Chapter 3 presents and analyses of the least-mean-square (LMS) algorithm in some depth. Several aspects are discussed, such as convergence behavior in stationary and nonstationary environments. This chapter also includes a number of theoretical as well as simulation examples to illustrate how the LMS algorithm performs in different setups. Appendix B addresses the quantization effects on the LMS algorithm when implemented in fixed- and floating-point arithmetics.

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Chapter 4 deals with some algorithms that are in a sense related to the LMS algorithm. In particular, the algorithms introduced are the quantized-error algorithms, the LMS-Newton algorithm, the normalized LMS algorithm, the transform-domain LMS algorithm, and the affine projection algorithm. Some properties of these algorithms are also discussed in Chapter 4, with special emphasis to the analysis of the affine projection algorithm.

Chapter 5 introduces the conventional recursive least-squares (RLS) algorithm. This algorithm minimizes a deterministic objective function, differing in this sense from most LMS-based algorithms. Following the same pattern of presentation of Chapter 3, several aspects of the conventional RLS algorithm are discussed, such as convergence behavior in stationary and nonstationary environments, along with a number of simulation results. Appendix C, deals with stability issues and quantization effects related to the RLS algorithm when implemented in fixed- and floating-point arithmetics. The results presented, except for the quantization effects, are also valid for the RLS algorithms presented in Chapters 7, 8, and 9. As a complement to Chapter 5, Appendix D presents the discrete-time Kalman filter formulation which despite being considered an extension of the Wiener filter has some relation with the RLS algorithm.

Chapter 6 discusses some techniques to reduce the overall computational complexity of adaptive filtering algorithms. The chapter first introduces the so called set-membership algorithms that update only when the output estimation error is higher than the prescribed upper bound. However, since set-membership algorithms require frequent updates during the early iterations in stationary environments, we introduce the concept of partial update to reduce the computational complexity in order to deal with situations where the available computational resources are not sufficient. This chapter presents several forms of set-membership algorithms related to the affine projection algorithms and their special cases. Chapter 6 also includes some simulation examples addressing standard as well as application oriented problems, where the algorithms of this and previous chapters are compared in some detail.

In Chapter 7, a family of fast RLS algorithms based on the FIR lattice realization is introduced. These algorithms represent interesting alternatives to the computationally complex conventional RLS algorithm. In particular, the unnormalized, the normalized and the error-feedback algorithms are presented.

Chapter 8 deals with the fast transversal RLS algorithms, which are very attractive due to their low computational complexity. However, these algorithms are known to face stability problems in practical implementation. As a consequence, special attention is given to the stabilized fast transversal RLS algorithm.

Chapter 9 is devoted to a family of RLS algorithms based on the QR decomposition. The conventional and a fast version of the QR-based algorithms are presented in this chapter.

Chapter 10 addresses the subject of adaptive filters using IIR digital filter realizations. The chapter includes a discussion on how to compute the gradient and how to derive the adaptive algorithms. The cascade, the parallel, and the lattice realizations are presented as interesting alternatives to the direct-form realization for the IIR adaptive filter. The characteristics of the mean-square error surface are also discussed in this chapter, for the IIR adaptive filtering case. Algorithms based on alternative error formulations, such as the equation error and Steiglitz-McBride methods are also introduced.

Chapter 11 deals with nonlinear adaptive filtering which consists of utilizing a nonlinear structure for the adaptive filter. The motivation is to use nonlinear adaptive filtering structures to better model some nonlinear phenomena commonly found in communications applications, such as nonlinear characteristics of power amplifier at transmitters. In particular, we introduce the Volterra series LMS and RLS algorithms, and the adaptive algorithms based on bilinear filters. Also, a brief introduction is given to some nonlinear adaptive filtering algorithms based on the concepts of neural networks, namely, the multilayer perceptron and the radial basis function algorithms. Some examples of DFE equalization are included in this chapter.

Chapter 12 deals with adaptive filtering in subbands mainly to address the applications where the required adaptive filter order is high, as for example in acoustic echo cancellation where the unknown system (echo) model has long impulse response. In subband adaptive filtering, some signals are split in frequency subbands via an analysis filter bank. Chapter 12 provides a brief review of multirate systems, and presents the basic structures for adaptive filtering in subbands. The concept of delayless subband adaptive filtering is also addressed, where the adaptive filter coefficients are updated in subbands and mapped to an equivalent fullband filter. The chapter also includes a discussion on the relation between subband and block adaptive filtering (also known as frequency-domain adaptive filters) algorithms.

Chapter 13 describes some adaptive filtering algorithms suitable for situations where no reference signal is available which are known as blind adaptive filtering algorithms. In particular, this chapter introduces some blind algorithms utilizing high-order statistics implicitly for the single-input single-output (SISO) equalization applications. In order to address some drawbacks of the SISO equalization systems, we discuss some algorithms using second-order statistics for the single-input multi-output (SIMO) equalization. The SIMO algorithms are naturally applicable in cases of oversampled received signal and multiple receive antennas. This chapter also discusses some issues related to blind signal processing not directly detailed here.

I decided to use some standard examples to present a number of simulation results, in order to test and compare different algorithms. This way, frequent repetition was avoided while allowing the reader to easily compare the performance of the algorithms. Most of the end of chapters problems are simulation oriented, however, some theoretical ones are included to complement the text.

The second edition differed from the first one mainly by the inclusion of chapters on nonlinear and subband adaptive filtering. Many other smaller changes were performed throughout the remaining chapters. In this edition, we introduced a number of derivations and explanations requested by students and suggested by colleagues. In addition, two new chapters on data-selective algorithms and blind adaptive filtering are included along with a large number of new examples and problems. Major changes take place in the first five chapters in order to make the technical details more accessible and to improve the ability of the reader in deciding where and how to use the concepts. The analysis of the fine projection algorithm is now presented in detail due to its growing practical importance. Several practical and theoretical examples are included aiming at comparing the families of algorithms introduced in the book.

In a trimester course, I usually cover Chapters 1 to 6 sometimes skipping parts of Chapter 2 and the analyses of quantization effects in Appendices B and C. In the remaining time, I try to cover as much as possible of the remaining chapters, usually consulting the audience to what they would

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prefer to study. This book can also be used for self-study where the reader can examine Chapters 1 to 6, and those not involved with specialized implementations can skip the Appendices B and C, without loss of continuity. The remaining chapters can be followed separately, except for Chapter 8 that requires reading Chapter 7. Chapters 7, 8, and 9 deal with alternative and fast implementations of RLS algorithms and the following chapters do not use their results.

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Prof. Paulo S. R. Diniz

Niterói, Brazil

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